First Semester M.Sc. Examination, January 2016
(CBCS)
MATHEMATICS
M 103 T : Topology – I

Time : 3 Hours Max. Marks : 70

Instructions : *i) Answer **any five full** questions.*
*ii) **All** questions carry **equal** marks.*

1. a) Define a finite and infinite sets. Let $g : X \to Y$ be an one-one correspondence. If the set X is infinite then prove that Y is infinite. **7**
   b) Prove that the set X is finite if and only if either $X = \phi$ or X is one-one correspondence with some N_k. **7**

2. a) State and prove Schroder-Bernstein theorem. **8**
   b) Define a denumerable set. Prove that every infinite subset of a denumerable set is denumerable. **6**

3. a) Let $A \subseteq (X, d)$ then prove the following :
      i) $X - A$ is open
      ii) $D(A) \subseteq A$, where D(A) is the derived set of A. **4**
   b) Prove that in a metric space (X, d), a closed ball is a closed set. **5**
   c) Prove that every metric space is a Hausdorff space. **5**

4. a) Show that a mapping $f : X \to Y$ is continuous if and only if $\{x_n\} \to x$ in X implies $\{f(x_n)\} \to f(x)$ in Y. **6**
   b) Define contraction mapping. Show that a contraction on a complete metric space has a unique fixed point. **8**

5. a) Show that every complete metric space is of the second category. **8**
   b) Prove that if f is an isometry, then f is a homeomorphism. **6**

6. a) Define a topology on a non-empty set. Prove that the intersection of two topologies is again a topology. Is the union of two topologies a topology ? Justify. **5**
   b) Prove that a set is closed if and only if it contains all its limit points. **5**
   c) Prove that a set A in $(X, \mathfrak{I})$ is open if and only if it is a neighbourhood of each of its points. **4**

7. a) Define interior, closure and boundary of a subset A of a topological space X. If A, B are any two subsets of X then prove the following : **7**

i) $X - A^{\circ} = \overline{X - A}$

ii) $b(A) = \overline{A} - A^{\circ}$

iii) $\overline{A} - \overline{B} \subset \overline{A - B}$

iv) $\overline{A \cup B} = \overline{A} \cup \overline{B}$.

b) Let $(X, \mathfrak{I})$ be a topological space and β a subfamily of $\mathfrak{I}$, then prove that β is a base for $\mathfrak{I}$ if and only if $U \in \mathfrak{I}$, $x \in U$ implies there is a B and β such that $x \in B \subseteq U$. **7**

8. a) Show that the mapping $f : X \to Y$ is continuous if and only if $\overline{f^{-1}(B)} \subseteq f^{-1}(\overline{B})$, $\forall B \subseteq Y$. **7**

b) Define a connected space. Give an example. Prove that the continuous image of a connected space is connected. **7**

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(CBCS)
MATHEMATICS
M 103T : Topology – I

Time : 3 Hours Max. Marks : 70

Instructions : *i) Answer **any five full** questions.*
*ii) **All** questions carry **equal** marks.*

1\. a) What do you mean by denumerable set ? Is Q, the set of rational numbers denumerable ? Justify. **4**

b) Show that :

i) Superset of an infinite set is infinite.

ii) Subset of a finite set is finite. **6**

c) Does every infinite set contains a denumerable ? If yes, explain. **4**

2\. a) State Schroder-Bernstein theorem.

Use it to prove that $(0, 1) \sim [0, 1]$. **3**

b) Let C = card R. Show that C.C = C. **6**

c) If P(A) denote the power set of a set A then prove that card A < card P(A). **5**

3\. a) Let (X, d) be a metric space and $d(x, y) = \dfrac{d(x, y)}{1 + d(x, y)}$.

Show that (X, d_1) is a metric space by checking only triangle inequality for d_1. **3**

b) Show that if a convergent sequence in a metric space has infinitely many distinct points then its limit is a limit point of the set of elements of the sequence. **4**

c) Prove that a subspace Y of a complete metric space is complete if it is closed. Is the converse true ? Explain. **7**

P.T.O.

4. a) Prove that if a metric space X is complete then for every nested sequence $\{F_n\}_1^\infty$ of a nonempty closed sets in X with $\delta(F_n) \to 0$, $\bigcap_{n=1}^{\infty} F_n$ is a singleton set. 6

b) Define a set of first category. State and prove Baire's category theorem. 8

5. a) Prove contraction mapping theorem. 6

b) Show that every metric space has a completion. 8

6. a) Show that intersection of two neighborhoods is a neighborhood. Is superset of a neighborhood is again a neighborhood ? Justify. 4

b) Show that :

i) an arbitrary intersection of closed sets is closed.

ii) a set containing all its limit points is closed. 6

c) Show that the interior of intersection of two sets is the intersection of their interiors, but this is not the case for the union of two sets. 4

7. a) Let (X, τ) be a topological space. Show that a subfamily $\mathscr{B}$ of τ is a base for τ if and only if for every $U \in \tau$ and $x \in U$ there is a $B \in \mathscr{B}$ such that $x \in B \subseteq U$. 6

b) Show that a function $f : X \to Y$ is continuous if and only if inverse of every open set in Y is open in X. 4

c) Show that a bijective function $f : X \to Y$ is a homeomorphism if and only if $f(\overline{A}) = \overline{f(A)}$, for all $A \subseteq X$. 4

8. a) If C is a connected subset of (X, Y) which has a separation $X = A \cup B$ then prove that either $C \subseteq A$ or $C \subseteq B$. 4

b) Show that closure of a connected set is connected. 5

c) Prove that union of family of connected sets with non-empty intersection. 5

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I Semester M.Sc. Degree Examination, January/February 2018
(CBCS Scheme)
MATHEMATICS
M103T : Topology – I

Time : 3 Hours

Max. Marks : 70

Instructions : *i) Answer **any five** questions.*
*ii) **All** questions carry **equal** marks.*

1. a) Let X be an infinite set and $x_0 \in X$, then prove that $X - \{x_0\}$ is infinite.

   b) Define countable set. Prove that a set X is infinite if and only if either $X = \phi$ or X is in one-to-one correspondence with some ¥$_k$, where ¥$_k = \{1, 2, 3, ..., k\}$ set of all natural numbers from 1 to k. **(6+8)**

2. a) Let X and Y be sets. If X is equivalent to a subset of Y and Y is equivalent to a subset of X, then prove that X and Y are equivalent.

   b) If P(A) denote the power set of a set A, then prove that card (P(A)) = $2^{card(A)}$. **(9+5)**

3. a) Define a metric space. If d is a metric on X, prove that $\rho(x, y) = \dfrac{d(x, y)}{1 + d(x, y)}$, $\forall x, y \in X$ is a metric on X.

   b) Prove that a subspace Y of complete metric space (X, d) is complete if and only if it is closed. **(7+7)**

4. a) State and prove contraction mapping theorem.

   b) State and prove Cantor's intersection theorem. **(6+8)**

5. a) Prove that an isometry is a homeomorphism but not conversely.

   b) Prove that every metric space has a completion. **(7+7)**

6. a) Prove that every metric space is a topological space.

   b) Prove that a set is open if and only if it is neighbourhood of each of its points.

   c) Prove that a point $x \in (X, \mathcal{J})$ belongs to the closure of a set A if and only if every open set G which contains x has a non-empty intersection with A. **(4+4+6)**

P.T.O.

7. a) Prove that $f : (X, \mathcal{J}) \to (Y, \mathcal{U})$ is continuous at $x \in X$ if and only if V is a neighbourhood of $f(x) \Rightarrow f^{-1}(V)$ is a neighbourhood of x.

   b) Prove that a bijective function $f : (X, \mathcal{J}) \to (Y, \mathcal{U})$ is a homeomorphism if and only if $f(A^0) = [f(A)]^0, \forall A \subseteq X$. (6+8)

8. a) Prove that a topological space $(X, \mathcal{J})$ is connected if and only if the only continuous map from X to the 2-point space over the constant map.

   b) Prove that the components of a totally disconnected space are its points. (7+7)